

Scale Dependence of the Halo Bias in General Local-Type Non-Gaussian Models I: Analytical Predictions and Consistency Relations

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Abstract. We investigate the clustering of halos in cosmological models starting with general local-type non-Gaussian primordial fluctuations. We employ multiple Gaussian fields and add local-type non-Gaussian corrections at arbitrary order to cover a class of models described by frequently-discussed f_{nl} , g_{nl} and τ_{nl} parameterization. We derive a general formula for the halo power spectrum based on the peak-background split formalism. The resultant spectrum is characterized by only two parameters responsible for the scale-dependent bias at large scale arising from the primordial non-Gaussianities in addition to the Gaussian bias factor. We introduce a new inequality for testing non-Gaussianities originating from multi fields, which is directly accessible from the observed power spectrum. We show that this inequality is a generalization of the Suyama-Yamaguchi inequality between f_{nl} and τ_{nl} to the primordial non-Gaussianities at arbitrary order. We also show that the amplitude of the scale-dependent bias is useful to distinguish the simplest quadratic non-Gaussianities (i.e., f_{nl} -type) from higher-order ones (g_{nl} and higher), if one measures it from multiple species of galaxies or clusters of galaxies. We discuss the validity and limitations of our analytic results by comparison with numerical simulations in an accompanying paper.

Keywords: large scale structure, primordial non-Gaussianity, numerical simulation

ArXiv ePrint: [1204.3490](https://arxiv.org/abs/1204.3490)

1 Introduction

The large-scale structure (LSS) of the universe as well as the temperature fluctuations of the cosmic microwave background (CMB) radiation are well understood in the standard framework of cosmological model starting from tiny almost Gaussian fluctuations. However, the generation mechanism of the primordial fluctuations, which seed both the CMB and the LSS, is yet to be understood. Because we usually consider that the primordial perturbations are created during inflation, their statistical properties provide us valuable information about the inflationary physics. Although we have no clear evidence of the violation of the Gaussian assumption of our initial condition to date, the signature of the primordial non-Gaussianities, if they are detected, can be a smoking gun to test various models of the primordial fluctuations in the coming era of precision cosmology.

Some proposed models of the early universe predict that large local-type non-Gaussianities are produced when nonlinear dynamics is important on super-horizon scales [1]. In these models, the Bardeen’s curvature potential Φ in the matter dominant era is written as a local function of a Gaussian field ϕ , and is usually expanded into a Taylor series as

$$\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{\text{nl}} [\phi^2(\mathbf{x}) - \langle \phi^2 \rangle] + g_{\text{nl}} \phi^3(\mathbf{x}) + h_{\text{nl}} [\phi^4(\mathbf{x}) - \langle \phi^4 \rangle] + i_{\text{nl}} \phi^5(\mathbf{x}) + \dots \quad (1.1)$$

The coefficient f_{nl} in the quadratic term is one of a key parameter to distinguish the models¹ [2]. This parameter determines the amplitude of the primordial bispectrum, the lowest-order statistic that captures the deviation from Gaussian statistics. In this model, the bispectrum of the curvature perturbation $\zeta \equiv (3/5)\Phi$ is written as

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\text{nl}} [P_\zeta(k_1)P_\zeta(k_2) + (2 \text{ perms.})], \quad (1.2)$$

where P_ζ denotes the power spectrum of ζ and (2 perms.) stands for two more terms given as permutations of wavenumbers, k_1 , k_2 and k_3 . The current constraints on the parameter f_{nl} from the CMB experiments are very tight, and any deviation from Gaussianity of order $\gtrsim 0.1\%$ level in the curvature perturbations has already been ruled out (e.g., [3]). It is expected that the constraint will be much tighter after some on-going/upcoming large observational projects such as Planck [4].

However, the value of f_{nl} does not determine all the statistical properties of the initial condition of our universe, even if one focuses only on the local-type non-Gaussian primordial fluctuations. The next order statistic, the trispectrum, is a natural extension of the discussion, and it indeed gives us more hints about the inflationary physics. The cubic-order correction to the curvature potential is a natural source of this statistic [5, 6] (g_{nl} -type; see Eq. 1.1). This gives the trispectrum of the form

$$T_\zeta(k_1, k_2, k_3, k_4) = \frac{54}{25} g_{\text{nl}} [P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3) + (3 \text{ perms.})]. \quad (1.3)$$

The trispectrum can also be generated through the quadratic corrections [7]. This type of trispectrum has a different scale dependence from Eq. (1.3) and is written as

$$T_\zeta(k_1, k_2, k_3, k_4) = \tau_{\text{nl}} [P_\zeta(k_1)P_\zeta(k_2)P_\zeta(|\mathbf{k}_1 + \mathbf{k}_3|) + (11 \text{ perms.})], \quad (1.4)$$

¹Note that we adopt the normalization of f_{nl} widely used in studies of the CMB, instead of the convention sometimes employed in the LSS works, where this parameter is also defined by Eq. (1.1), but the normalization of the curvature potential is done at the present by linear extrapolation.

where the parameter τ_{nl} controls the amplitude of the spectrum. In single-field models described by Eq. (1.1), τ_{nl} can be expressed by f_{nl} as $\tau_{\text{nl}} = (36/25)f_{\text{nl}}^2$. In general, however, when one employs two or more Gaussian components in the primordial fluctuations, τ_{nl} can be larger, and the following inequality is shown to be always valid ([8]; Suyama-Yamaguchi inequality):

$$\tau_{\text{nl}} \geq \frac{36}{25}f_{\text{nl}}^2, \quad (1.5)$$

where f_{nl} is defined not by Eq. (1.1) but by Eq. (1.2) as the amplitude of the bispectrum of the curvature perturbations. See [9–11] for further discussions on the loop corrections and the universality of this inequality, and [12] for an observational attempt to test it from existing observations. Thanks to this inequality, one can distinguish multi-field inflationary models from single-field ones by measuring f_{nl} and τ_{nl} simultaneously. The statistical properties of the primordial perturbations are uniquely determined by three parameters, f_{nl} , g_{nl} and τ_{nl} , up to the order of trispectrum as long as one is interested in local-type non-Gaussianities², and a number of models can be sorted by these parameters [9].

The LSS that we observe through the clustering of galaxies or clusters of galaxies is a highly non-Gaussian field due to the nonlinear gravitational evolution (e.g., [15] for a review), redshift-space distortions caused by the peculiar velocity field of the galaxies [16] and the galaxy bias with respect to the underlying matter density field [17]. These acquired non-Gaussianities are thought to prevent us to measure only the primordial non-Gaussianity from the LSS unlike the CMB, whose temperature fluctuations are still in the linear stage. It is thus not straightforward to extract the non-Gaussian signature of purely primordial origin from the LSS, and a number of studies have been done to model the gravitationally generated non-Gaussianities [18–22]. In these studies, the authors mainly focus on the galaxy bispectrum, simply because it is the lowest-order statistic that captures the non-Gaussian signature (but see [20] for a discussion on the trispectrum).

However, one observable feature in the LSS, which was realized only recently, attracts a great attention [23]. That is the *scale-dependent bias* in the halo power spectrum on large scales (i.e., $\gtrsim \text{Gpc}$). This is an interesting example where the bias, which usually prevents us from extracting cosmological information, opens a new window to see the primordial bispectrum but in the power spectrum. This new feature has been extensively studied both analytically [24–31] and numerically [32–34], and gives stringent constraints on f_{nl} [35–37], which are already competitive to those from the CMB. Also, the scale-dependent bias affects the halo bispectrum similarly, and thus there is a possibility to put even stronger constraints on f_{nl} from the LSS by combining the power spectrum with the bispectrum [34, 38–41].

A similar scale-dependent halo bias has been reported in the presence of the primordial trispectrum (see [42–45] for g_{nl} -type and [45–47] for τ_{nl} -type). Furthermore, the scale dependence in the halo bias is found to be present in other models of the primordial non-Gaussianities, such as the scale-dependent f_{nl} model [48, 49], ungaussiton model [50], and non-local non-Gaussian models [51–53]. Thus the interpretation is not straightforward even when one detects a clear evidence of the scale-dependent bias from observed clustering of galaxies or clusters of galaxies.

Our aim is to develop a statistical methodology to distinguish these models. Moreover, we would like to generalize the argument as model-independent as possible. In this paper, as a first attempt, we focus on the scale-independent local-type non-Gaussianities and discuss

²Note, however, that these parameters can be scale dependent in general (see, e.g., [13, 14]).

the halo clustering. In particular, we employ multiple Gaussian fields that seed the curvature perturbations in order to control f_{nl} and τ_{nl} independently. We also keep all the higher-order corrections in the curvature perturbations such as g_{nl} and h_{nl} to see how generic our result is. Our model is a simple generalization of the f_{nl} , g_{nl} and τ_{nl} parameterization. We will show that this class of primordial non-Gaussianities result in a universal formula for the halo power spectrum. We introduce a new parameterization of the non-Gaussian signature in the scale-dependent bias and propose two tests to distinguish different models based on the analytic results. The validity and the limitations of our analytical results are tested by confronting with a large set of cosmological N -body simulations, and will be presented in a separate paper (hereafter paper II).

This paper is organized as follows. We first introduce our model of the primordial non-Gaussianities and discuss the basic statistical properties in Sec. 2. We then compute the halo power spectrum following the peak-background split argument in Sec. 3. We discuss the two relevant parameters for the feature in the scale-dependent bias and show how they are useful to distinguish different models in Sec. 4. We give a brief summary of the paper in Sec. 5. Throughout the paper, we adopt the best-fit flat Λ CDM cosmology to the seven-year observations of WMAP [3] and adopt $A_s = 2.43 \times 10^{-9}$, $n_s = 0.963$ for the amplitude and tilt of the power spectrum of the curvature perturbations normalized at wavenumber $k_0 = 0.002 \text{ Mpc}^{-1}$, $\Omega_m = 0.265$, $\Omega_b = 0.0448$ for the density parameters of total matter and baryon and $h = 0.71$ for the Hubble parameter. We compute the transfer function from the curvature perturbation to the matter density by CAMB [54] using these parameters.

2 The model

In this section, we describe our model of the non-Gaussian curvature perturbations and show their basic statistical properties. This section gives useful quantities to derive the clustering of halos in later sections, such as the cumulants of the density field. We first introduce our parameterization for the curvature perturbations in Sec. 2.1, and compute the polyspectra (Sec. 2.2) and cumulants (Sec. 2.3).

2.1 Local-type non-Gaussianities with two fields

In this subsection, we introduce our models of the non-Gaussian primordial perturbations. Throughout the paper, we restrict our attention to the adiabatic density fluctuations, whose statistical properties are solely transferred from the primordial curvature perturbations:

$$\delta_{\mathbf{k}}(z) = \mathcal{M}(k, z)\zeta_{\mathbf{k}}, \quad (2.1)$$

where $\mathcal{M}(k, z)$ denotes the matter transfer function, and is normalized such that $\delta_{\mathbf{k}}(z)$ gives the linear overdensity field at redshift z . We hereafter omit the redshift dependence, but it always comes through $\mathcal{M}(k, z)$. Then, we consider the local-type non-Gaussianities in the curvature perturbations originating from two Gaussian fields. We expand the curvature perturbations into a Taylor series:

$$\zeta(\mathbf{x}) = \chi_1(\mathbf{x}) + \chi_2(\mathbf{x}) + \sum_{i,j} c_{i,j} \left[\chi_1^i(\mathbf{x}) \chi_2^j(\mathbf{x}) - \langle \chi_1^i \chi_2^j \rangle \right], \quad (2.2)$$

where χ_1 and χ_2 are two statistically independent auxiliary Gaussian fields, $\langle \chi_1 \chi_2 \rangle = 0$. In the above, the indices i and j in the summation run over positive integers with $i + j \geq 2$. We

assume that the linear terms in Eq. (2.2) are dominant contributions and the higher-order terms give little corrections to the total curvature perturbations.

After the main discussion starting with the two-field model in Eq. (2.2) we will show how the result can be generalized to multi-field models with $N \geq 2$ independent Gaussian fields. Our final results for the halo power spectrum do not depend on the number of fields as will be shown in Sec. 3.2.

2.2 Polyspectra

Let us first compute the polyspectra based on Eq. (2.2). We focus on models in which the two fields, χ_1 and χ_2 , have the same spectral index, and parameterize their power spectra as

$$P_{\chi_1}(k) = (1 - \alpha) P_\zeta(k), \quad (2.3)$$

$$P_{\chi_2}(k) = \alpha P_\zeta(k), \quad (2.4)$$

$$P_\zeta(k) \equiv \frac{2\pi^2}{k^3} A_s \left(\frac{k}{k_0} \right)^{n_s-1}, \quad (2.5)$$

Note that Eq. (2.2) reduces to the single-field model (1.1) when $\alpha = 0$. We can generalize the model (2.2) into scale-dependent non-Gaussianities by setting different tilts for the two fields, and these models will be discussed elsewhere.

We then consider the higher-order polyspectra of ζ at the tree-level. The model (2.2) reproduces the bispectrum of the curvature perturbations expressed by Eq. (1.2), with the amplitude parameter f_{nl} being the following form:

$$f_{\text{nl}} = \frac{5}{3} [(1 - \alpha)^2 c_{2,0} + \alpha(1 - \alpha) c_{1,1} + \alpha^2 c_{0,2}]. \quad (2.6)$$

Similarly, the model gives the trispectrum written as the sum of Eqs. (1.3) and (1.4), and the relevant parameters read

$$g_{\text{nl}} = \frac{25}{9} [(1 - \alpha)^3 c_{3,0} + \alpha(1 - \alpha)^2 c_{2,1} + \alpha^2(1 - \alpha) c_{1,2} + \alpha^3 c_{0,3}], \quad (2.7)$$

$$\tau_{\text{nl}} = 4(1 - \alpha)^3 c_{2,0}^2 + \alpha(1 - \alpha) c_{1,1}^2 + 4\alpha^3 c_{0,2}^2 + 4\alpha(1 - \alpha) c_{1,1} [(1 - \alpha) c_{2,0} + \alpha c_{0,2}]. \quad (2.8)$$

The trispectrum has two terms, one comes from the cubic coupling, $c_{i,j}$ with $i + j = 3$, and is characterized by g_{nl} , while the other originates from $c_{i,j}$ with $i + j = 2$ (i.e., the τ_{nl} term). If one specifies the values of f_{nl} , g_{nl} and τ_{nl} , the primordial trispectrum as well as the bispectrum are uniquely determined.

In this paper, we further discuss the primordial non-Gaussianities beyond the trispectrum level to confirm the generality of the conclusion. In order to do so, we compute the tetraspectrum (the fifth order spectrum) of the curvature perturbations. In addition to the contribution from h_{nl} in Eq. (1.1), we find two more terms arising from lower-order non-Gaussianities (i.e., f_{nl} and g_{nl}). They are shown in Appendix A. As a specific example, we investigate the effect of quartic-order non-Gaussianity characterized by h_{nl} , and the results of N -body simulations will also be presented in paper II.

2.3 Cumulants

The cumulants of the density field are basic quantities useful in analytically estimating the halo number density (see Sec. 3.2 below). In this subsection, we compute the low-order cumulants and give their fitting formulae.

We begin by defining the smoothed density field in Fourier space:

$$\delta_{R,\mathbf{k}} \equiv \widetilde{W}_R(k) \delta_{\mathbf{k}}, \quad (2.9)$$

where \widetilde{W}_R is the Fourier transform of the window function at scale R , and we adopt the top hat function. We then define its n -th order cumulant (or, the connected moment) as

$$\bar{\kappa}_R^{(n)} \equiv \langle \delta_R^n \rangle_c. \quad (2.10)$$

We introduce the windowed transfer function, $\mathcal{M}_R(k) \equiv \widetilde{W}_R(k) \mathcal{M}(k)$, which directly connects the smoothed density contrast to the curvature perturbation:

$$\delta_{R,\mathbf{k}} = \mathcal{M}_R(k) \zeta_{\mathbf{k}}. \quad (2.11)$$

Using Gaussianity of χ_1 and χ_2 , the leading order non-vanishing cumulants are computed as

$$\bar{\kappa}_R^{(n)} = \sum_{i_1, \dots, i_n, j_1, \dots, j_n} \left[\prod_{m=1}^n c_{i_m, j_m} \right] \left\langle \prod_{m=1}^n \left[\mathcal{M}_R * \left(\chi_1^{i_m} \chi_2^{j_m} - \langle \chi_1^{i_m} \chi_2^{j_m} \rangle \right) \right] \right\rangle_c, \quad (2.12)$$

where the product $*$ represents a convolution with the windowed transfer function:

$$(\mathcal{M}_R * A)(\mathbf{x}) = \int d^3\mathbf{y} \mathcal{M}_R(\mathbf{x} - \mathbf{y}) A(\mathbf{y}), \quad (2.13)$$

with $\mathcal{M}_R(\mathbf{x})$ being the Fourier counterpart of $\mathcal{M}_R(k)$. In Eq. (2.12), the summation runs for $2n$ non-negative integers, i_m and j_m , which satisfy

$$i_m + j_m \geq 1, \quad \text{for } m = 1, 2, \dots, n, \quad (2.14)$$

$$\sum_{m=1}^n (i_m + j_m) = 2(n-1). \quad (2.15)$$

They can be explicitly rewritten by integrals of the polyspectra of ζ , already computed in the previous subsection, after multiplying the window function. For example, the first three cumulants, the variance $\bar{\kappa}_R^{(2)}$, the skewness $\bar{\kappa}_R^{(3)}$ and the kurtosis $\bar{\kappa}_R^{(4)}$, are given by

$$\bar{\kappa}_R^{(2)} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \mathcal{M}_R(p)^2 P_\zeta(p) \equiv \bar{\sigma}_R^2, \quad (2.16)$$

$$\bar{\kappa}_R^{(3)} = \int \frac{d^3\mathbf{p} d^3\mathbf{q}}{(2\pi)^6} \mathcal{M}_R(p) \mathcal{M}_R(q) \mathcal{M}_R(|\mathbf{p} + \mathbf{q}|) B_\zeta(p, q, |\mathbf{p} + \mathbf{q}|) \equiv f_{\text{nl}} \hat{\kappa}_{R, f_{\text{nl}}}^{(3)}, \quad (2.17)$$

$$\begin{aligned} \bar{\kappa}_R^{(4)} &= \int \frac{d^3\mathbf{p} d^3\mathbf{q} d^3\mathbf{r}}{(2\pi)^9} \mathcal{M}_R(p) \mathcal{M}_R(q) \mathcal{M}_R(r) \mathcal{M}_R(|\mathbf{p} + \mathbf{q} + \mathbf{r}|) T_\zeta(\mathbf{p}, \mathbf{q}, \mathbf{r}, -\mathbf{p} - \mathbf{q} - \mathbf{r}), \\ &\equiv g_{\text{nl}} \hat{\kappa}_{R, g_{\text{nl}}}^{(4)} + \tau_{\text{nl}} \hat{\kappa}_{R, \tau_{\text{nl}}}^{(4)}, \end{aligned} \quad (2.18)$$

where $\hat{\kappa}_{R, g_{\text{nl}}}^{(4)}$ and $\hat{\kappa}_{R, \tau_{\text{nl}}}^{(4)}$ originate from the trispectrum in Eq. (1.3) and (1.4), respectively. We find that the following fitting formulae for the skewness and kurtosis accurately reproduce the numerical integrals of the above equations for our fiducial cosmology at $z = 0$:

$$\hat{\kappa}_{R, f_{\text{nl}}}^{(3)} \simeq 3.20 \times 10^{-4} \bar{\sigma}_R^{3.141}, \quad (2.19)$$

$$\hat{\kappa}_{R, g_{\text{nl}}}^{(4)} \simeq 5.81 \times 10^{-8} \bar{\sigma}_R^{4.297}, \quad (2.20)$$

$$\hat{\kappa}_{R, \tau_{\text{nl}}}^{(4)} \simeq 2.06 \times 10^{-7} \bar{\sigma}_R^{4.288}. \quad (2.21)$$

See also [55] and [56] for similar fitting formulae. We also derive analogous expression for the fifth-order cumulant, $\bar{\kappa}_R^{(5)}$. It can be found in Appendix A.

3 Analytical prediction of the halo clustering

So far, we have focused on the basic properties of the underlying density field as well as the curvature perturbations. Now, let us consider a simple model of the halos in the presence of general local-type non-Gaussianities described by Eq. (2.2).

We follow the peak-background split formalism, and generalize it to our model, Eq. (2.2) in Sec. 3.1. There, we define the conditional cumulants and show how these cumulants are affected by primordial non-Gaussianities. After that, we discuss the clustering properties of halos based on the calculation of the conditional number density of them in Sec 3.2.

3.1 The peak-background split in the presence of non-Gaussianities

3.1.1 long and short mode decomposition

The peak-background split formalism had been originally introduced to model the biased clustering of peaks, halos, or galaxies which are associated with the initial peak regions in the Lagrangian space [57–60]. This formalism gives a simple framework to compute the bias of the collapsed objects with mass M by considering the *conditional* probability that a density contrast with a mode of scale R exceeds a critical density δ_c under the background density field modulated by long-wavelength modes. Here, the mass and the scale are related as

$$R \equiv \left(\frac{3M}{4\pi\bar{\rho}} \right)^{1/3}, \quad (3.1)$$

where $\bar{\rho}$ denotes the cosmic mean density. Effectively, the presence of long modes can be interpreted as a local shift in the threshold density:

$$\delta_c \rightarrow \delta_c(\mathbf{q}) \equiv \delta_c - \delta_\ell(\mathbf{q}), \quad (3.2)$$

where the coordinate \mathbf{q} indicates that we are working in the Lagrangian space. We adopt $\delta_c = 1.686$ following the result of the spherical collapse model, and leave the discussion on its more accurate modeling to paper II. Using Eq. (3.2), one can relate the long mode to the halo number density at that position with a help of the halo mass function.

In the presence of primordial non-Gaussianities, Eq. (3.2) is not the only effect from the long modes, since Fourier modes are not independent of each other from the beginning. We will show how this mode coupling results in the statistical properties of the short modes that are responsible for the formation of halos.

We start with decomposition of the two auxiliary Gaussian fields into sums of the long and short wavelength contributions:

$$\chi_i(\mathbf{q}) = \chi_{i,\ell}(\mathbf{q}) + \chi_{i,s}(\mathbf{q}), \quad i = 1, 2. \quad (3.3)$$

Then, keeping only the linear modulation from $\chi_{i,\ell}$, the short mode for the density fluctuation can be computed as

$$\delta_s(\mathbf{q}) = \sum_{i,j} [c_{i,j} + \Delta c_{i,j}(\mathbf{q})] (\mathcal{M}_R * \chi_{1,s}^i \chi_{2,s}^j)(\mathbf{q}), \quad (3.4)$$

$$\Delta c_{i,j}(\mathbf{q}) \equiv (i+1)c_{i+1,j} \chi_{1,\ell}(\mathbf{q}) + (j+1)c_{i,j+1} \chi_{2,\ell}(\mathbf{q}), \quad (3.5)$$

where $c_{1,0} = c_{0,1} = 1$ and the convolution $*$ is defined by Eq. (2.13). Interestingly, one can see that a $(n+1)$ th-order coupling constant in ζ (i.e., $c_{i,j}$ with $i+j = n+1$) modifies δ_s at

the n th order. For instance, in case of quadratic non-Gaussianities characterized by f_{nl} , the short mode is affected by the long mode at the linear order (i.e., the amplitude of the power spectrum is modulated), and this gives the scale-dependent bias [35]. We will see how the statistical properties of the conditional density field (3.4) result in the characteristic feature in the halo clustering shortly.

3.1.2 conditional cumulants

In Sec. 2.3, we have computed the cumulants of the density field in the presence of primordial non-Gaussianities. The cumulants, denoted by $\bar{\kappa}_R^{(n)}$, are given as the volume averages of the products of the density contrast over the entire universe. One of the key idea in the peak-background split formalism is to consider the *conditional* cumulants of the short mode at an arbitrary position \mathbf{q} in the presence of the long modes. These cumulants are defined as the ensemble average of δ_R^n while keeping the long mode at that Lagrangian position (e.g., $\chi_{i,\ell}(\mathbf{q})$ in our case) fixed:

$$\kappa_R^{(n)}(\mathbf{q}) \equiv \langle \delta_R^n \rangle_{c,s|\ell}. \quad (3.6)$$

Then, we can easily show how the long mode, which couples with the short mode due to the primordial non-Gaussianities, modify the cumulants of the short mode locally.

In Eq. (3.4), we can see that the nonlinear coupling constant in the short mode of the density contrast can be obtained by replacements of the coupling constants to the effective ones

$$c_{i,j} \rightarrow c_{i,j}(\mathbf{q}) \equiv c_{i,j} + \Delta c_{i,j}(\mathbf{q}), \quad (3.7)$$

which vary from position to position proportional to a certain combination of $\chi_{1,\ell}$ and $\chi_{2,\ell}$. We can obtain the conditional cumulants by applying the above replacements to Eq. (2.12) derived for the unconditional cumulants. Let us consider first the conditional variance of the short mode fluctuations of the density contrast. We define the local rms fluctuation, $\sigma_R(\mathbf{q}) \equiv [\kappa_R^{(2)}(\mathbf{q})]^{1/2}$, and this is given by

$$\sigma_R(\mathbf{q}) = \left\{ 1 + [2(1 - \alpha)c_{2,0} + \alpha c_{1,1}] \chi_{1,\ell}(\mathbf{q}) + [(1 - \alpha)c_{1,1} + 2\alpha c_{0,2}] \chi_{2,\ell}(\mathbf{q}) \right\} \bar{\sigma}_R, \quad (3.8)$$

where we keep only linear corrections from $\chi_{1,\ell}$ and $\chi_{2,\ell}$. Similarly, we can compute the higher-order conditional cumulants of the short mode when the long modes are fixed. Again, we keep the leading-order corrections arising from $\chi_{i,\ell}$, and obtain

$$\kappa_R^{(3)}(\mathbf{q}) = [f_{\text{nl}} + \Delta f_{\text{nl}}(\mathbf{q})] \hat{\kappa}_{R,f_{\text{nl}}}^{(3)}, \quad (3.9)$$

$$\kappa_R^{(4)}(\mathbf{q}) = [g_{\text{nl}} + \Delta g_{\text{nl}}(\mathbf{q})] \hat{\kappa}_{R,g_{\text{nl}}}^{(4)} + [\tau_{\text{nl}} + \Delta \tau_{\text{nl}}(\mathbf{q})] \hat{\kappa}_{R,\tau_{\text{nl}}}^{(4)}, \quad (3.10)$$

for the skewness and kurtosis. Here we define the effective shifts in the coupling constants, $\Delta f_{\text{nl}}(\mathbf{q})$, $\Delta g_{\text{nl}}(\mathbf{q})$ and $\Delta \tau_{\text{nl}}(\mathbf{q})$. They can be explicitly written down as linear combinations

of $\chi_{1,\ell}(\mathbf{q})$ and $\chi_{2,\ell}(\mathbf{q})$:

$$\Delta f_{\text{nl}}(\mathbf{q}) = \frac{5}{3} \left\{ \left[(1-\alpha)^2 (4c_{2,0}^2 + 3c_{3,0}) + \alpha(1-\alpha) (2c_{1,1}c_{2,0} + c_{1,1}^2 + 2c_{2,1}) + \alpha^2 (2c_{0,2}c_{1,1} + c_{1,2}) \right] \chi_{1,\ell}(\mathbf{q}) + \left[1 \leftrightarrow 2 \right] \chi_{2,\ell}(\mathbf{q}) \right\}, \quad (3.11)$$

$$\Delta g_{\text{nl}}(\mathbf{q}) = \frac{25}{9} \left\{ \left[2(1-\alpha)^3 (3c_{2,0}c_{3,0} + 2c_{4,0}) + \alpha(1-\alpha)^2 (4c_{2,0}c_{2,1} + c_{1,1}c_{2,1} + 3c_{3,1}) + 2\alpha^2 (1-\alpha) (c_{2,0}c_{1,2} + c_{1,1}c_{1,2} + c_{2,2}) + \alpha^3 (3c_{1,1}c_{0,3} + c_{1,3}) \right] \chi_{1,\ell}(\mathbf{q}) + \left[1 \leftrightarrow 2 \right] \chi_{2,\ell}(\mathbf{q}) \right\}, \quad (3.12)$$

$$\Delta \tau_{\text{nl}}(\mathbf{q}) = \left\{ \left[8(1-\alpha)^3 (2c_{2,0}^3 + 3c_{2,0}c_{3,0}) + 4\alpha(1-\alpha)^2 (2c_{2,0}c_{1,1}^2 + 2c_{2,0}^2c_{1,1} + c_{1,1}c_{2,1} + 3c_{1,1}c_{3,0} + 2c_{2,0}c_{2,1}) + 2\alpha^2 (1-\alpha) (c_{1,1}^3 + 4c_{2,0}c_{1,1}c_{0,2} + 2c_{1,1}^2c_{0,2} + 2c_{1,1}c_{2,1} + 4c_{0,2}c_{2,1} + 2c_{1,1}c_{1,2}) + 8\alpha^3 (c_{1,1}c_{0,2}^2 + c_{0,2}c_{1,2}) \right] \chi_{1,\ell}(\mathbf{q}) + \left[1 \leftrightarrow 2 \right] \chi_{2,\ell}(\mathbf{q}) \right\}, \quad (3.13)$$

where the coefficients for $\chi_{2,\ell}(\mathbf{x})$, denoted as $[1 \leftrightarrow 2]$, can be obtained by performing permutations $c_{m,n} \leftrightarrow c_{n,m}$ and $\alpha \leftrightarrow (1-\alpha)$ to the corresponding coefficients of $\chi_{1,\ell}(\mathbf{x})$. See also Appendix B for the results for single-field non-Gaussianities (1.1).

An important point here is that the n th-order correction in Eq. (2.2) modifies the n th-order conditional cumulant, while the unconditional cumulant at the same order is not affected. For instance, if one is interested in quadratic models with $f_{\text{nl}} \neq 0$, the first non-trivial contribution to the conditional cumulants appears in the variance, while the unconditional variance does not depend on f_{nl} . Similarly, when $g_{\text{nl}} \neq 0$, the conditional skewness has a linear dependence on the parameter g_{nl} (see also [30, 43]), while this parameter gives the amplitude of the unconditional kurtosis.

3.2 Halo number density and clustering

The halo mass function is known to be almost universal and solely determined by the parameter $\nu \equiv \delta_c/\sigma_R$, where δ_c denotes the critical density above which halos form at $z = 0$ [61]. The mass function is usually characterized by $\bar{f}(M, z)$:

$$\bar{n}(M, z) \equiv \frac{d\mathcal{N}}{dM} = \bar{f}(M, z) \frac{\bar{\rho}}{M^2} \left| \frac{d \ln \bar{\sigma}_R^{-1}}{d \ln M} \right|, \quad (3.14)$$

where we denote by $\bar{n}(M, z)$ the number density of halos with mass M at redshift z , and $\bar{\rho}$ is the cosmic mean density [62]. When the halo mass function is universal, the function $\bar{f}(M, z)$ depends only on the peak height ν . However, as we are interested in corrections from the higher-order cumulants, we slightly relax the assumption of the universality. We allow a dependence on the density cumulants at any order and write

$$\bar{f}(M, z) = \bar{f} \left(\bar{\sigma}_R, \bar{\kappa}_R^{(3)}, \bar{\kappa}_R^{(4)}, \dots; \delta_c \right). \quad (3.15)$$

Here, the argument R has a one-to-one correspondence to the halo mass M via Eq. (3.1).

We then assume that the halo number density field is a local process in Lagrangian space: given the conditional cumulants defined in the previous subsection, we can deterministically

predict the number of halos at that (Lagrangian) position. More specifically, we assume that the halo number density is a function of the conditional cumulants smoothed with the physical scale corresponding to the halo mass in the Lagrangian space. We introduce $f(\mathbf{q}|M, z)$ analogously to its *unconditional* counterpart, \bar{f} , in Eq. (3.14), and assume that this function can be obtained by replacements of the arguments in \bar{f} as

$$\bar{\kappa}_R^{(n)} \rightarrow \kappa_R^{(n)}(\mathbf{q}), \quad (3.16)$$

in addition to the change in the critical density for the formation of halos in Eq. (3.2). We have

$$f(\mathbf{q}|M, z) = \bar{f}\left(\sigma_R(\mathbf{q}), \kappa_R^{(3)}(\mathbf{q}), \kappa_R^{(4)}(\mathbf{q}), \dots; \delta_c - \delta_\ell(\mathbf{q})\right). \quad (3.17)$$

This function gives us a simple route to the halo power spectrum. The halo density contrast in the Lagrangian space can be calculated as

$$\delta_h^L(\mathbf{q}|M, z) = \frac{f(\mathbf{q}|M, z)}{\bar{f}(M, z)} - 1. \quad (3.18)$$

By expanding this equation into Taylor series of δ_ℓ , $\chi_{1,\ell}$ and $\chi_{2,\ell}$, and collecting all the leading-order contributions, we obtain the linear halo overdensity field in the Lagrangian space. We then map this field to the Eulerian space. At leading order, we have

$$\delta_h(\mathbf{x}) = b_\delta \delta(\mathbf{x}) + b_{\chi_1} \chi_1(\mathbf{x}) + b_{\chi_2} \chi_2(\mathbf{x}), \quad (3.19)$$

with coefficients defined through the derivatives of the mass function:

$$b_\delta = 1 - \frac{\partial \ln f}{\partial \delta_c}, \quad (3.20)$$

$$b_{\chi_i} = \frac{\partial \ln f}{\partial \ln \sigma_R} \frac{\partial \ln \sigma_R}{\partial \chi_{i,\ell}} + \sum_{n=3}^{\infty} \frac{\partial \ln f}{\partial \kappa_R^{(n)}} \frac{\partial \kappa_R^{(n)}}{\partial \chi_{i,\ell}}, \quad i = 1, 2. \quad (3.21)$$

In Eq. (3.19), we dropped the subscript ℓ for notational convenience. Note that the first term in Eq. (3.20) comes from the mapping to the Eulerian space. Note also that the derivatives of the conditional cumulants with respect to $\chi_{i,\ell}$ can be written down explicitly in terms of $c_{i,j}$ and α by using the expressions derived in Sec. 3.1.2. Taking the average of the square of Eq. (3.19) in Fourier space, we finally arrive at the halo power spectrum:

$$P_h(k) = b_\delta^2 P_\delta(k) + 2 r_{\text{MF}} b_\delta b_\zeta P_{\delta\zeta}(k) + b_\zeta^2 P_\zeta(k), \quad (3.22)$$

where the density and density-curvature power spectra are defined as

$$P_\delta(k) \equiv \mathcal{M}^2(k) P_\zeta(k), \quad P_{\delta\zeta}(k) \equiv \mathcal{M}(k) P_\zeta(k), \quad (3.23)$$

and we parameterize the bias coefficients by

$$b_\zeta \equiv \sqrt{(1 - \alpha) b_{\chi_1}^2 + \alpha b_{\chi_2}^2}, \quad (3.24)$$

$$r_{\text{MF}} \equiv \frac{(1 - \alpha) b_{\chi_1} + \alpha b_{\chi_2}}{\sqrt{(1 - \alpha) b_{\chi_1}^2 + \alpha b_{\chi_2}^2}}. \quad (3.25)$$

As we will discuss later in more detail, the parameter b_ζ characterizes the amplitude of the non-Gaussian corrections to the halo power spectrum. On the other hand, the other parameter r_{MF} captures the shape of the scale dependent bias, and plays a key role to distinguish the models for the primordial non-Gaussianities based on multi fields from those based on a single field.

It is interesting to see that a wide class of models described by Eq. (2.2) end up with the same result for the halo power spectrum with only two parameters responsible for non-Gaussian corrections (b_ζ and r_{MF}) in addition to the usual (Gaussian) bias factor, b_δ . Moreover, our result can be generalized to models with more than two independent fields very easily. If we employ N statistically independent Gaussian fields, χ_n , and expand the curvature perturbations as

$$\zeta(\mathbf{x}) = \sum_n \chi_n(\mathbf{x}) + \sum_{i_1, \dots, i_N} c_{i_1, \dots, i_N} \left[\prod_n \chi_n^{i_n}(\mathbf{x}) - \left\langle \prod_n \chi_n^{i_n} \right\rangle \right], \quad (3.26)$$

the halo density contrast of Eq. (3.19) becomes

$$\delta_h(\mathbf{x}) = b_\delta \delta(\mathbf{x}) + \sum_n b_{\chi_n} \chi_n(\mathbf{x}), \quad (3.27)$$

with the index n running from unity to N . From this equation, it is clear that Eq. (3.22) still holds as long as all the N fields have the same spectral index. In this case, the bias coefficients are given by

$$b_\zeta \equiv \sqrt{\sum_n \alpha_n b_{\chi_n}^2}, \quad (3.28)$$

$$r_{\text{MF}} \equiv \frac{\sum_n \alpha_n b_{\chi_n}}{\sqrt{\sum_n \alpha_n b_{\chi_n}^2}}, \quad (3.29)$$

with the fraction in the power spectrum amplitude of the n -th field denoted as α_n that satisfy

$$\sum_n \alpha_n = 1. \quad (3.30)$$

Equation (3.27) can be regarded as a generalization of the multi-variate biasing in case of the single-field models discussed in [28].

We show an example halo power spectrum in Fig. 1. We can see how each of the three terms in Eq. (3.22) contributes to the total power spectrum. The non-Gaussian corrections are prominent on large scales ($\lesssim 0.01 h\text{Mpc}^{-1}$). The parameters, b_ζ and r_{MF} as well as b_δ can be determined from observed power spectrum with a sufficient survey volume thanks to the different k -dependence of the three terms. We propose to use these parameters as direct observables from the measurements of the scale-dependent halo bias instead of popular non-Gaussian parameters such as f_{nl} and g_{nl} , because the latter set of parameters have strong degeneracy in many cases. We will see how our new parameterization helps us to test different class of models for the primordial non-Gaussianities in the next section.

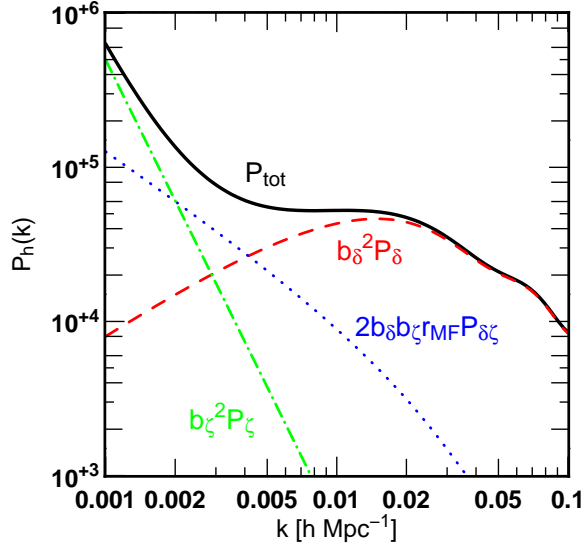


Figure 1. Example plot of the halo power spectrum (Eq. 3.22) in case of $b_\delta = 2$, $b_\zeta = 100$ and $r_{\text{MF}} = 1$. We plot the total power spectrum by solid line, while the three terms in that equation are respectively shown in dashed, dotted and dot-dashed lines.

3.3 Recovery of previous studies

It is worth comparing our results with the predictions in the literature. In the presence of the primordial non-Gaussianities based on a single field, it is straightforward to show that $r_{\text{MF}} = 1$ or -1 (see Sec. 4.1 for more discussion on r_{MF}). Then, Eq. (3.22) can be rewritten as

$$P_h(k) = [b_\delta + \Delta b(k)]^2 P_\delta(k), \quad (3.31)$$

$$\Delta b(k) \equiv \text{sign}(r_{\text{MF}}) b_\zeta \mathcal{M}^{-1}(k), \quad (3.32)$$

where we denote the sign of A as $\text{sign}(A)$. In this case, one can obtain the scale-dependent bias in the halo power spectrum simply by replacing a constant bias factor with a k -dependent one: $b_\delta \rightarrow b_\delta + \Delta b(k)$. All the effects from the primordial non-Gaussianities are encoded in b_ζ , the amplitude of the scale-dependent bias. This parameter in case of the single-field primordial non-Gaussianities (1.1) is explicitly given in Appendix B.

Let us further restrict the model to the quadratic non-Gaussianities (i.e., f_{nl} -type). We find that the first term in Eq. (3.21) is the dominant contribution to b_ζ , and the derivatives of cumulants $\kappa_R^{(n)}$ with $n \geq 3$ are small. By dropping these small corrections, and using that the derivative with respect to δ_c in Eq. (3.20) can be converted to that with respect to σ_R under the assumption of the universal halo mass function, we recover

$$\Delta b(k) = \frac{6}{5} f_{\text{nl}} \delta_c (b_\delta - 1) \mathcal{M}^{-1}(k), \quad (3.33)$$

derived by [23]. The correction terms from higher-order cumulants might be important if one needs a very accurate modeling for the scale-dependent bias. However, since these terms strongly depends on the non-Gaussian mass function and we adopted a rather simplified one described in Appendix C in this paper, we left further discussions on the accurate modeling in paper II.

We next consider cubic non-Gaussianities described by g_{nl} . In this case, the first term of Eq. (3.21) is zero, and thus the dominant effect comes from the derivative of the mass function with respect to the skewness. Using the expression for the conditional skewness given in Appendix B, and dropping the corrections from higher-order cumulants, we have

$$\Delta b(k) = \frac{9}{5} g_{\text{nl}} \hat{\kappa}_{f_{\text{nl}}, R}^{(3)} \frac{\partial \ln f}{\partial \ln \kappa_R^{(3)}} \mathcal{M}^{-1}(k), \quad (3.34)$$

This is identical to Ref. [43]. One can recover the formula for the scale-dependent bias derived in [30, 31] by substituting the mass function based on the Edgeworth expansion to this expression (see Appendix C). Indeed, Ref. [43] demonstrated that the N -body data can be accurately explained by evaluating the derivative of the mass function directly from simulations.

Let us put some comments on non-Gaussianities based on multi-field initial conditions. Refs. [46, 47] compute the clustering of halos in a two-field inflationary model using the peak-background split. They employ two fields, one is Gaussian and the other is non-Gaussian with a quadratic correction (i.e., f_{nl} -type), and thus our model (2.2) includes their model. However, their main focus is on the stochasticity between the halo and the underlying matter density fields. Although our derivation in this paper is equivalent to theirs, our focus is more on the auto-power spectrum of halos. It is straightforward to show that the stochasticity arises between the halo and the matter fields in our models when $\alpha \neq 0$ regardless of the order of the non-Gaussian corrections, and this is another unique feature to test multi-field non-Gaussianities.

Note also that our analytical calculation of the halo power spectrum is valid only at the linear order. Although we are interested in the clustering on large scales (say, $\gtrsim 1 h^{-1} \text{Gpc}$) and linear theory is expected to work well, any nonlinearity (e.g., nonlinear gravitational growth or nonlinear bias) can in principle modify our results. For example, the nonlinearity in the halo bias can source a similar scale dependence as shown in [24] (and see also [30] for a comparison among different derivations of the scale-dependent bias including the effect of nonlinear bias in the thresholding scheme). As another example, the matter power spectrum is affected by a loop correction in case of g_{nl} -type models, which does not vanish at large scale limit [42]. Another important assumption in our analysis is the locality of the formation of halos in Lagrangian space. These issues will be discussed in paper II with fully nonlinear treatment based on cosmological N -body simulations.

4 Testing non-Gaussian models with observation

We have shown that our model (3.26) generally results in the halo power spectrum Eq. (3.22). The final result of the halo power spectrum contains only three parameters, b_δ , b_ζ and r_{MF} , and they can be determined or constrained directly from the observation using the different k -dependence of the three terms in Eq. (3.22). Following this result, we discuss what properties of the primordial non-Gaussianities can be tested by measurements of these parameters.

4.1 Cauchy-Schwartz inequality and multi-field non-Gaussianities

We discuss what we can learn from the parameter r_{MF} defined in Eq. (3.29). By simple calculation, one can show that r_{MF} satisfies the following inequality:

$$|r_{\text{MF}}| \leq 1, \quad (4.1)$$

where the equality holds when the curvature perturbations originate from a single field. In multi-field models, the above equality holds only when all the coefficients b_{χ_i} are the same. Note that this inequality is the Cauchy-Schwartz inequality among Gaussian fields, χ_i .

Measurement of this parameter opens a new window to distinguish between single-field and multi-field primordial non-Gaussianities. If the absolute value of r_{MF} measured from the observations of the galaxy clustering significantly deviates from unity, we can rule out single-field models of the primordial fluctuations. Interestingly, this inequality is shown to be identical to the Suyama-Yamaguchi inequality in case of the quadratic non-Gaussianities under some approximations. Let us focus on two-field models (2.2) for simplicity. As we have already mentioned, the dominant contribution to the bias coefficients in Eq. (3.21) is the first term in the presence of quadratic non-Gaussianities, $c_{i,j}$ with $i + j = 2$. If we drop the higher-order corrections, we have

$$b_{\chi_1} \approx [2(1 - \alpha)c_{2,0} + \alpha c_{1,1}] \frac{\partial \ln f}{\partial \ln \sigma_R}, \quad (4.2)$$

$$b_{\chi_2} \approx [(1 - \alpha)c_{1,1} + 2\alpha c_{0,2}] \frac{\partial \ln f}{\partial \ln \sigma_R}. \quad (4.3)$$

Substituting these expressions into Eq. (3.25), we finally have

$$r_{\text{MF}} \approx \frac{6f_{\text{nl}}}{5\sqrt{\tau_{\text{nl}}}}, \quad (4.4)$$

with a help of the definitions of f_{nl} and τ_{nl} in Eqs. (2.6) and (2.8). Note that Eq. (4.4) still holds in more general case of Eq. (3.26). Thus, we can directly test the Suyama-Yamaguchi inequality from the observed power spectrum. Note that our parameter r_{MF} is equivalent to $A_{\text{NL}}^{-1/2}$ introduced in [12] under this approximation. Since our inequality is valid for models with higher-order non-Gaussianities, it can be regarded as a generalization of the Suyama-Yamaguchi inequality, and can be directly tested from the observed halo power spectrum.

The parameter r_{MF} controls the shape of the scale dependent bias. This is illustrated in Fig. 2. In the left panel, we show how the halo power spectrum depends on r_{MF} when b_δ and b_ζ are fixed. The solid lines correspond to models of primordial non-Gaussianities originating from a single-field with $f_{\text{nl}} = \pm 100$, while the dashed lines show the halo power spectra from multi-field non-Gaussian initial conditions. We also plot by symbols the expected error bars from a $100h^{-3}\text{Gpc}^3$ survey at $z = 1$. We employ the formula by [63] for the $1\text{-}\sigma$ error bars that is valid for Gaussian fields:

$$[\Delta P_g(k)]^2 = \frac{1}{N_{\text{mode}}} \left[P_g(k) + \frac{1}{n_g} \right]^2, \quad (4.5)$$

where N_{mode} denotes the number of independent Fourier modes. The fiducial model shown as the error bars in the left panel corresponds to $f_{\text{nl}} = 20$ and $\tau_{\text{nl}} = 14,400$, if we assume that the non-Gaussianities are quadratic. The value of f_{nl} is small enough to satisfy the current observational bounds, while that of τ_{nl} is much larger than $(36/25)f_{\text{nl}}^2$, which might be possible to detect from future observations.

From the figure, we can see the largest difference among the five lines at $k \sim 0.005h\text{Mpc}^{-1}$, where the $P_{\delta\zeta}$ term in Eq. (3.22) has the largest impact. On the other hand, they converge at the small- k limit since we adopt the same value of b_ζ for them. We need to measure the broadband shape of the power spectrum covering this entire scale at $\gtrsim h^{-1}\text{Gpc}$ to obtain

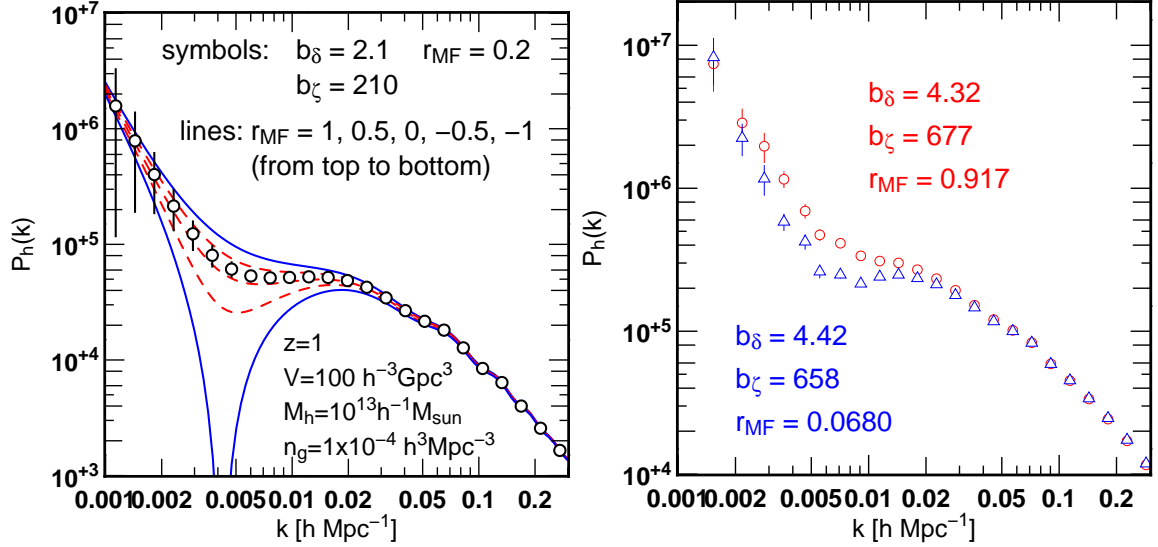


Figure 2. Dependence on the shape parameter r_{MF} of the halo power spectrum. Left: analytical results of the r_{MF} dependence at $z = 1$. Lines show models with different r_{MF} while $b_\delta = 2.1$ and $b_\zeta = 210$ are fixed. Expected error bars on the galaxy power spectrum from a $100 h^{-3} \text{Gpc}^3$ class survey are also shown assuming $r_{\text{MF}} = 0.2$ as the fiducial value. We assume that the typical host halo mass is $10^{13} h^{-1} M_\odot$ (this gives the value of b_δ shown in the figure), and the number density of halos is $n_h = 10^{-4} h^3 \text{Mpc}^{-3}$. Right: examples for halo power spectra measured from N -body simulations. We fit the data by model (3.22) to find the best-fit parameters, b_δ , b_ζ and r_{MF} . They are shown in the figure for each model.

meaningful constraints on r_{MF} . The planned surveys such as EUCLID [64] can measure the galaxy power spectrum quite accurately even at such a large scale thanks to their huge survey volumes. They will provide great opportunities for this kind of tests.

We also show in the right panel of Fig. 2 the measured halo power spectra from N -body simulations at $z = 1$. We plot the results obtained in simulations starting with two different non-Gaussian initial conditions; a single-field model (circles), and a two-field model (triangles). The input parameters of these simulations are $c_{2,0} = 60$ and $\alpha = 0$ for the single-field model, and $c_{2,0} = 120$, $c_{0,2} = -120$ and $\alpha = 1/2$ for the two-field model. The derived non-Gaussian parameters for the two simulations are $(f_{\text{nl}}, \tau_{\text{nl}}) = (100, 14400)$ and $(0, 14400)$, respectively. Thus, the two simulations are expected to have roughly the same values for b_δ and b_ζ . We fit the measured halo power spectrum by Eq. (3.22) to obtain b_δ , b_ζ and r_{MF} , which are shown in the panel. The best-fit value of r_{MF} in case of the single-field model shown in the figure is consistent with unity within the statistical uncertainty, as expected. On the other hand, the multi-field model clearly disfavors $|r_{\text{MF}}| = 1$. These results basically confirm our analytical expectations. See paper II for more detail on the simulations and analyses.

It is interesting to note that in Ref. [45], the authors reach a similar conclusion for testing f_{nl} and τ_{nl} through the shape of the halo power spectrum but from a different derivation based on the clustering of peaks above a threshold. Our results agree with theirs qualitatively. One advantage of our formulation is that our inequality (4.1) is always valid with non-Gaussian corrections at an arbitrary order and can be used to test multi-field models for the primordial fluctuations in general.

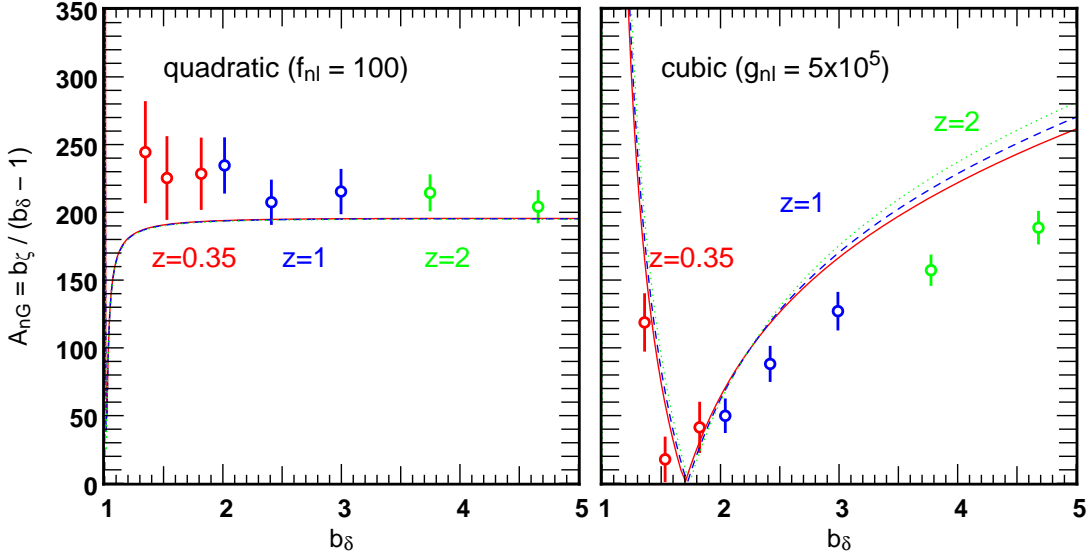


Figure 3. Dependence on the Gaussian bias factor, b_δ , of the non-Gaussian correction to the halo bias. We plot the combination $A_{\text{nG}} = b_\zeta / (b_\delta - 1)$ as a function of b_δ at redshifts 0.35, 1 and 2. The error bars show the $1\text{-}\sigma$ uncertainty obtained by fitting to N -body simulations, while the lines depict our theoretical predictions at $z = 0.35$ (solid), $z = 1$ (dashed) and $z = 2$ (dotted). The left (right) panel plots the results for quadratic (cubic) non-Gaussianities characterized by f_{nl} (g_{nl}).

4.2 Mass dependence and the order of the non-Gaussian corrections

We finally discuss how we should distinguish the quadratic non-Gaussianities (i.e., f_{nl} -type) from higher-order models. As we have shown in the previous subsection, the shape of the halo (or galaxy) power spectrum on large scales only gives the information about the number of independent fields through the parameter r_{MF} . However, the f_{nl} -type non-Gaussianities are distinguishable from higher-order ones if one examines more than one galaxy populations with different bias parameters, in principle. In this section, we discuss how the ratio

$$A_{\text{nG}}(M) \equiv \frac{b_\zeta(M)}{b_\delta(M) - 1}, \quad (4.6)$$

can be used for this test.

For simplicity, let us focus on the primordial curvature perturbations originating from a single Gaussian field (1.1). Figure 3 shows how the amplitude of the non-Gaussian correction to the halo bias scales with the Gaussian bias factor for halo catalogs with different masses at different redshifts. We plot A_{nG} measured from N -body simulations (error bars) and estimated by our analytic calculation (lines). We adopt two models for the primordial non-Gaussianities, and show the results for $f_{\text{nl}} = 100$ in the left panel and $g_{\text{nl}} = 5 \times 10^5$ in the right panel.

As expected from Eq. (3.33) derived in the literature, this ratio remains almost constant over the Gaussian bias factor (or the halo mass) and redshift in case of quadratic non-Gaussianities. In contrast, the scaling of A_{nG} with b_δ observed in the right panel is very different. A clear increasing tendency is visible from both simulations (symbols) and analytical predictions (lines). Interestingly, the redshift dependence of the relation between A_{nG} and b_δ is very weak similarly to the left panel for the f_{nl} -type primordial non-Gaussianities.

Since we need an accurate modeling of the derivatives of the mass function with respect to the cumulants to compute the analytical curves, it is generally difficult to reach a quantitative agreement with simulations. See Appendix C for more detail of the mass function we adopt in computing the analytical predictions.

Although our analytical calculation has a slight offset to the N -body data, the qualitative agreement with the N -body simulations is rather encouraging. The difference between the two models shown in Fig. 3 originates from the different origins of the scale-dependent bias. In the presence of a non-zero f_{nl} , most of the effect comes from the conditional variance as was shown in the previous section. In this case, b_ζ roughly scales as $\propto (b_\delta - 1)$, and thus A_{nG} does not depend on the halo mass. In contrast, we find that the origin of the scale-dependent bias is in the conditional skewness in the presence of non-zero g_{nl} .

Using this feature, we propose the following test. Measure the power spectrum from multiple tracers, such as blue and red galaxies, massive and less massive halos at any redshifts. Then, fit by Eq. (3.22) to obtain b_ζ as well as b_δ and compare the ratio A_{nG} measured from each tracer. If the two (or more) values of A_{nG} are inconsistent with statistical significance, that might be a sign of the primordial non-Gaussianities which originate from higher-order coupling (i.e., g_{nl} or even higher). We should have a constant A_{nG} independent of the properties of the tracers in case of the quadratic non-Gaussianities characterized by f_{nl} . We also examine the parameter A_{nG} in the presence of quartic-order non-Gaussianities described by h_{nl} (see Eq. 1.1). We find that the b_δ dependence of this parameter is similar to what is found in the presence of cubic non-Gaussianities. Thus, we conclude that the constancy of A_{nG} is generally a key feature for discriminating f_{nl} -type non-Gaussianities from higher-order models.

5 Summary and Discussions

We computed the halo power spectrum in a class of local-type non-Gaussianities described by Eq. (3.26) based on the peak-background split formalism. We found that the resultant halo power spectrum generally exhibits a scale dependence in the large scale bias factor and is parameterized by three parameters; the ordinary Gaussian bias factor, b_δ , the shape parameter of the scale dependence, r_{MF} , and the amplitude of the scale dependence, b_ζ . We showed how one can distinguish between non-Gaussianities based on a single field and multi fields through the measurement of r_{MF} . This allows us a direct test of the generalized Suyama-Yamaguchi inequality (4.1) from observational data. Another parameter, b_ζ might be useful to distinguish f_{nl} -type models from g_{nl} -type or models with even higher-order non-Gaussianities. The parameter A_{nG} , a combination of b_ζ and b_δ , is shown to be almost constant in case of f_{nl} -type non-Gaussianities, though it can have a strong dependence on the mass of halos in general. Thus, we propose to measure this parameter from multiple tracers to search for a possible dependence of A_{nG} on the halo mass to test higher-order non-Gaussian models.

There are several concerns on the systematic error and possible degeneracy with other models for the primordial non-Gaussianities. Since our analytic model is based on the leading-order calculation, any loop-correction can be a source of systematic error. This can be a problem for both of our two tests, if the correction is large. We thus have to test our model against simulations. A detailed comparison with N -body simulations will be presented in a separate paper.

Another systematic effect, though it is avoidable by a careful theoretical modeling, is induced from a mass dependence in the critical density for the halo formation, so called the moving barrier [65]. According to the moving barrier argument, the critical density δ_c might not be a constant, and should be replaced by $\delta_c(M)$ with an explicit halo-mass dependence. In the presence of non-zero f_{nl} , the ratio A_{nG} can be approximated as

$$A_{\text{nG}}(M) \approx \frac{6}{5} f_{\text{nl}} \delta_c(M), \quad (5.1)$$

when we neglect terms from higher-order cumulants. This equation implies that the mass dependence in $\delta_c(M)$ can be misinterpreted as a sign of higher-order non-Gaussianities. We will discuss in paper II how this matters using N -body simulations, and show that in the presence of non-zero f_{nl} , we have no clear evidence of the scale dependence in $\delta_c(M)$.

The halo assembly bias is another source of systematics. Ref. [66] showed that the amplitude of the scale-dependent bias for a given halo mass can be different when they divide the halo sample into subsamples with different halo assembly histories. This can be a problem when the observed galaxies form in halos with a certain merging history preferentially. The problem of non-Gaussian halo assembly bias may also be interpreted as a dependence on the assembly history of the barrier function $\delta_c(M)$. We need a careful modeling of the barrier function as a function of the assembly history in addition to the mass.

Finally, scale dependent f_{nl} models can also result in a mass dependent ratio A_{nG} . In Ref. [49], the authors discuss the scale-dependent bias in models with k dependence in the coupling parameter $f_{\text{nl}}(k)$. They show that the magnitude of the non-Gaussian correction to the bias in the power spectra of massive and less massive halos depends on $f_{\text{nl}}(k)$ evaluated at wavenumber roughly corresponding to the mass scale of the halos (see also [30, 31, 48]). Thus, this dependence can mimic the mass dependence of A_{nG} that we found in the presence of higher-order non-Gaussianities. However, by detecting a mass dependence of this parameter, we might be able to falsify the simplest non-Gaussian model with a constant f_{nl} , and thus this test is still interesting.

This work is the first step towards establishing a methodology to test non-standard primordial non-Gaussianities beyond the so-called f_{nl} type in the era of $100h^{-3}\text{Gpc}^3$ class full sky surveys. We expect that our tests for the multi-field non-Gaussianities as well as the higher-order ones will be possible with future surveys such as EUCLID.

Acknowledgments

We appreciate Atsushi Taruya for careful reading of the manuscript and giving us valuable suggestions and comments. We thank Fabian Schmidt for comments on the derivative terms in the non-Gaussian halo mass functions. We also thank Masahide Yamaguchi for useful comments. T. N. is supported by a Grant-in-Aid for Japan Society for the Promotion of Science (JSPS) Fellows (PD: 22-181) and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. Numerical computations for the present work have been carried out in part on Cray XT4 at Center for Computational Astrophysics, CfCA, of National Astronomical Observatory of Japan, and in part under the Interdisciplinary Computational Science Program in Center for Computational Sciences, University of Tsukuba.

A Fifth-order statistics

In this appendix, we show the relevant formulae for fifth-order statistics in model (3.26). In actual calculation of the bias coefficients (3.21) in the text, we truncate the series at the fifth-order cumulants. The numerical results shown in Fig. 3 are based on the formulae in this section.

First, the tetraspectrum of the curvature perturbations reads

$$\begin{aligned} T_\zeta^{(5)}(\mathbf{k}_1, \dots, \mathbf{k}_5) = & \frac{648}{125} h_{\text{nl}} [P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + (4 \text{ perms.})] \\ & + \tau_{\text{nl}}^{(\text{II})} [P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)P_\zeta(|\mathbf{k}_1 + \mathbf{k}_4|) + (59 \text{ perms.})] \\ & + \tau_{\text{nl}}^{(\text{III})} [P_\zeta(k_1)P_\zeta(k_2)P_\zeta(|\mathbf{k}_1 + \mathbf{k}_3|)P_\zeta(|\mathbf{k}_2 + \mathbf{k}_4|) + (59 \text{ perms.})]. \end{aligned} \quad (\text{A.1})$$

In the above, the coefficients, $\tau_{\text{nl}}^{(\text{II})}$ and $\tau_{\text{nl}}^{(\text{III})}$ are respectively given as

$$\tau_{\text{nl}}^{(\text{II})} = \frac{324}{125} f_{\text{nl}} g_{\text{nl}}, \quad \tau_{\text{nl}}^{(\text{III})} = \frac{216}{125} f_{\text{nl}}^3, \quad (\text{A.2})$$

when ζ originates from a single field as in Eq. (1.1), and they can be different in multi-field cases. Corresponding to the three terms for the tetraspectrum, we give the fitting formulae for the fifth-order cumulant of the smoothed density field at $z = 0$:

$$\bar{\kappa}_R^{(5)} = h_{\text{nl}} \hat{\kappa}_{R, h_{\text{nl}}}^{(5)} + \tau_{\text{nl}}^{(\text{II})} \hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{II})}}^{(5)} + \tau_{\text{nl}}^{(\text{III})} \hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{III})}}^{(5)}, \quad (\text{A.3})$$

$$\hat{\kappa}_{R, h_{\text{nl}}}^{(5)} = 1.26 \times 10^{-11} \bar{\sigma}_R^{5.456}, \quad (\text{A.4})$$

$$\hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{II})}}^{(5)} = 7.82 \times 10^{-11} \bar{\sigma}_R^{5.385}, \quad (\text{A.5})$$

$$\hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{III})}}^{(5)} = 9.67 \times 10^{-11} \bar{\sigma}_R^{5.288}. \quad (\text{A.6})$$

The fifth-order conditional cumulants are summarized in the next appendix.

B Exact expressions for single-field non-Gaussianities

We mainly focus on non-Gaussian curvature perturbations based on multi fields in the text. It might be convenient to show the relevant formulae for the single field model (1.1).

We start with the conditional cumulants. First, substituting $\alpha = 0$ and with a help of Eq. (2.6), the conditional variance of the short-mode fluctuations in Eq. (3.8) is obtained as:

$$\sigma_R(\mathbf{q}) = \left[1 + \frac{6}{5} f_{\text{nl}} \zeta_\ell(\mathbf{q}) \right] \bar{\sigma}_R, \quad (\text{B.1})$$

where we use the fact that $\zeta = \chi_1$ at linear order. This is equivalent to the results of earlier works in case of f_{nl} -type non-Gaussianities (e.g., [28, 35]). We then write down the shift in the effective nonlinear parameters Eqs. (3.11), (3.12) and (3.13). Substituting $\alpha = 0$, they are simplified as

$$\Delta f_{\text{nl}}(\mathbf{q}) = \frac{3}{5} (4f_{\text{nl}}^2 + 3g_{\text{nl}}) \zeta_{G, \ell}(\mathbf{q}), \quad (\text{B.2})$$

$$\Delta g_{\text{nl}}(\mathbf{q}) = \frac{6}{5} (3f_{\text{nl}} g_{\text{nl}} + 2h_{\text{nl}}) \zeta_{G, \ell}(\mathbf{q}), \quad (\text{B.3})$$

$$\Delta \tau_{\text{nl}}(\mathbf{q}) = \frac{216}{125} f_{\text{nl}} (2f_{\text{nl}}^2 + 3g_{\text{nl}}) \zeta_{G, \ell}(\mathbf{q}). \quad (\text{B.4})$$

We can obtain similar expressions for the fifth-order cumulant, $\kappa_R^{(5)}$. This is given as

$$\kappa_R^{(5)}(\mathbf{q}) = [h_{\text{nl}} + \Delta h_{\text{nl}}(\mathbf{q})] \hat{\kappa}_{R, h_{\text{nl}}}^{(5)} + [\tau_{\text{nl}}^{(\text{II})} + \Delta \tau_{\text{nl}}^{(\text{II})}(\mathbf{q})] \hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{II})}}^{(5)} + [\tau_{\text{nl}}^{(\text{III})} + \Delta \tau_{\text{nl}}^{(\text{III})}(\mathbf{q})] \hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{III})}}^{(5)} \quad (\text{B.5})$$

where

$$\Delta h_{\text{nl}}(\mathbf{q}) = \frac{3}{5} (8f_{\text{nl}} h_{\text{nl}} + 5i_{\text{nl}}) \zeta_{\text{G}, \ell}(\mathbf{q}), \quad (\text{B.6})$$

$$\Delta \tau_{\text{nl}}^{(\text{II})}(\mathbf{q}) = \frac{972}{625} (4f_{\text{nl}} h_{\text{nl}} + 3g_{\text{nl}}^2 + 6f_{\text{nl}}^2 g_{\text{nl}}) \zeta_{\text{G}, \ell}(\mathbf{q}), \quad (\text{B.7})$$

$$\Delta \tau_{\text{nl}}^{(\text{III})}(\mathbf{q}) = \frac{648}{625} f_{\text{nl}}^2 (9g_{\text{nl}} + 4f_{\text{nl}}^2) \zeta_{\text{G}, \ell}(\mathbf{q}). \quad (\text{B.8})$$

Given the formulae for the conditional cumulants up to the fifth order, it is straightforward to derive the bias coefficient, b_ζ . It is

$$\begin{aligned} \text{sign}(r_{\text{MF}}) b_\zeta &= \frac{6}{5} f_{\text{nl}} \frac{\partial \ln f}{\partial \ln \sigma_R} + \frac{3}{5} (4f_{\text{nl}}^2 + 3g_{\text{nl}}) \hat{\kappa}_{R, f_{\text{nl}}}^{(3)} \frac{\partial \ln f}{\partial \kappa_R^{(3)}} \\ &+ \left[\frac{6}{5} (3f_{\text{nl}} g_{\text{nl}} + 2h_{\text{nl}}) \hat{\kappa}_{R, g_{\text{nl}}}^{(4)} + \frac{216}{125} f_{\text{nl}} (2f_{\text{nl}}^2 + 3g_{\text{nl}}) \hat{\kappa}_{R, \tau_{\text{nl}}}^{(4)} \right] \frac{\partial \ln f}{\partial \kappa_R^{(4)}} \\ &+ \left[\frac{3}{5} (8f_{\text{nl}} h_{\text{nl}} + 5i_{\text{nl}}) \hat{\kappa}_{R, h_{\text{nl}}}^{(5)} + \frac{972}{625} (4f_{\text{nl}} h_{\text{nl}} + 3g_{\text{nl}}^2 + 6f_{\text{nl}}^2 g_{\text{nl}}) \hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{II})}}^{(5)} \right. \\ &\left. + \frac{648}{625} f_{\text{nl}}^2 (9g_{\text{nl}} + 4f_{\text{nl}}^2) \hat{\kappa}_{R, \tau_{\text{nl}}^{(\text{III})}}^{(5)} \right] \frac{\partial \ln f}{\partial \kappa_R^{(5)}}, \end{aligned} \quad (\text{B.9})$$

where the mass function is give in the next appendix.

C Non-Gaussian halo mass function

We need knowledge of the mass function to evaluate the bias coefficients. In this paper, we assume a simple mass function and that is described in this appendix.

We assume that the primordial non-Gaussianities are small, and expand the non-Gaussian probability distribution function around normal distribution to obtain Edgeworth series as in [67]:

$$\begin{aligned} \frac{\text{PDF}_{\text{nG}}(\nu)}{\text{PDF}_{\text{G}}(\nu)} &= 1 + \frac{1}{6} \overline{C}_R^{(3)} H_3(\nu) \\ &+ \frac{1}{24} \overline{C}_R^{(4)} H_4(\nu) + \frac{1}{72} \left(\overline{C}_R^{(3)} \right)^2 H_6(\nu) \\ &+ \frac{1}{120} \overline{C}_R^{(5)} H_5(\nu) + \frac{1}{144} \overline{C}_R^{(3)} \overline{C}_R^{(4)} H_7(\nu) + \frac{1}{1296} \left(\overline{C}_R^{(3)} \right)^3 H_9(\nu) + \dots, \end{aligned} \quad (\text{C.1})$$

where $H_n(\nu)$ is the n -th order Hermite polynomial, and we introduce normalized cumulants, $\overline{C}_R^{(n)} \equiv \bar{\kappa}_R^{(n)} / \bar{\sigma}_R^n$. Since we are interested in the primordial non-Gaussianities arising from the coupling in the curvature perturbations higher than the quadratic order characterized by f_{nl} , we take account of the cumulants up to the fifth, $\bar{\kappa}_R^{(5)}$, such that we can discuss the effect from the quartic correction, h_{nl} . We thus keep the series up to the fourth order in terms

of the Edgeworth expansion (see Eq. C.1). We follow the argument by [67] and obtain the non-Gaussian halo mass function relevant to this order:

$$\begin{aligned}\frac{\bar{f}(M, z)}{\bar{f}_G(M, z)} &= \frac{\text{PDF}_{\text{nG}}(\nu)}{\text{PDF}_G(\nu)} + \frac{1}{6} \frac{d\bar{C}_R^{(3)}}{d \ln \bar{\sigma}_R} \frac{1}{\nu} H_2(\nu) + \frac{1}{24} \frac{d\bar{C}_R^{(4)}}{d \ln \bar{\sigma}_R} \frac{1}{\nu} H_3(\nu) \\ &+ \frac{1}{72} \frac{d(\bar{C}_R^{(3)})^2}{d \ln \bar{\sigma}_R} \frac{1}{\nu} H_5(\nu) + \frac{1}{120} \frac{d\bar{C}_R^{(5)}}{d \ln \bar{\sigma}_R} \frac{1}{\nu} H_4(\nu) \\ &+ \frac{1}{144} \frac{d(\bar{C}_R^{(3)} \bar{C}_R^{(4)})}{d \ln \bar{\sigma}_R} \frac{1}{\nu} H_6(\nu) + \frac{1}{1296} \frac{d(\bar{C}_R^{(3)})^3}{d \ln \bar{\sigma}_R} \frac{1}{\nu} H_8(\nu),\end{aligned}\quad (\text{C.2})$$

where \bar{f}_G denotes the Gaussian mass function. Note that as long as we consider the local-type non-Gaussianities, the derivative terms of $\bar{C}_R^{(n)}$ are usually very small given their weak $\bar{\sigma}_R$ dependence (see Eqs. 2.19, 2.20, 2.21, and also Eqs. A.4, A.5, A.6), and the first term gives the dominant correction. Note also that these derivative terms result in the correction terms in the bias parameters derived by [30, 31].

Although the above ratio is derived based on the Press-Schechter formalism [62], we simply replace the Gaussian mass function with one recently calibrated by a large set of N -body simulations by [68]:

$$\bar{f}_G(M, z) = f_{\text{MICE}}(\bar{\sigma}_R, z) = A(z) \left[\bar{\sigma}_R^{-a(z)} + b(z) \right] \exp \left[-\frac{c(z)}{\bar{\sigma}_R^2} \right], \quad (\text{C.3})$$

where $A(z) = 0.58(1+z)^{-0.13}$, $a(z) = 1.37(1+z)^{-0.15}$, $b(z) = 0.3(1+z)^{-0.084}$ and $c(z) = 1.036(1+z)^{-0.024}$. This mass function is shown to agree with N -body simulations over five orders of magnitude in the halo mass.

Although the mass function we adopt in this paper to incorporate the effect of the primordial non-Gaussianities might be too simplistic, one can easily refine the final result by replacing the mass function to a more accurate one. See e.g., [29, 32, 33, 42, 69–71] for various elaborated halo mass functions for non-Gaussian initial conditions.

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